Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2223–7214

19 pages

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Zaha is designing a bridge to cross a river. She believes that the weight of the steel needed for this bridge is approximately 53 632 000 kg.

The exact weight of the steel needed for the bridge is 55 625 000 kg.

(a) Find the percentage error in Zaha's approximation. [2]

Zaha's design is used to build five identical bridges.

(b) (i) Find the weight of the steel needed for these five bridges, **to three significant figures**.

(ii) Write down your answer to part (b)(i) in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. [3]

[Maximum mark: 6]

Angel has $ 520 in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

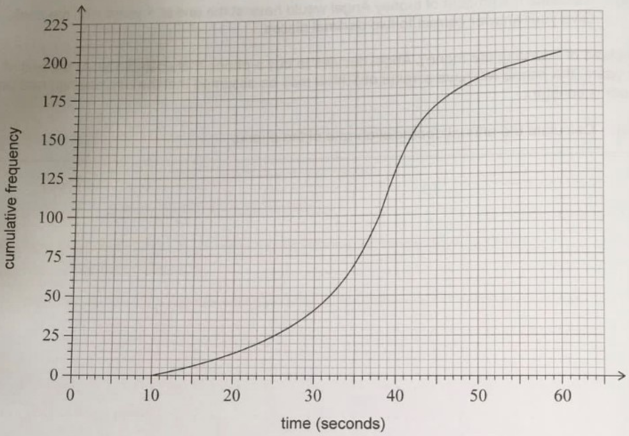
(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs $ 520. At the end of 5 years, the phone will have a value of $ 30. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

3. [Maximum mark: 7]

In a school, 200 students solved a problem in a mathematics competition. Their times to solve the problem were recorded and the following cumulative frequency graph was produced.



(a) Use the graph to find

(i) the median time;

(ii) the lower quartile;

(iii) the upper quartile;

(iv) the interquartile range. [4]

Cedric took 14 seconds to solve the problem.

(b) Determine whether Cedric's time is an outlier. [3]

(This question continues on the following page)

4. [Maximum mark: 6]

At a running club, Sung-Jin conducts a test to determine if there is any association between an athlete's age and their best time taken to run $100\,\text{m}$. Eight athletes are chosen at random, and their details are shown below.

Athlete	A	B	C	D	E	F	G	H
Age (years)	13	17	22	18	19	25	11	36
Time (seconds)	13.4	14.6	13.4	12.9	12.0	11.8	17.0	13.1

Sung-Jin decides to calculate the Spearman's rank correlation coefficient for his set of data.

(a) Complete the table of ranks. [2]

Athlete	A	B	C	D	E	F	G	H
Age rank			3					
Time rank							1	

(b) Calculate the Spearman's rank correlation coefficient, r_s. [2]

(c) Interpret this value of r_s in the context of the question. [1]

(d) Suggest a mathematical reason why Sung-Jin may have decided not to use Pearson's product-moment correlation coefficient with his data from the original table. [1]

5. [Maximum mark: 4]

The following frequency distribution table shows the test grades for a group of students.

Grade	1	2	3	4	5	6	7
Frequency	1	4	7	9	p	9	4

For this distribution, the mean grade is 4.5.

(a) Write down the total number of students in terms of p. [1]

(b) Calculate the value of p. [3]

6. [Maximum mark: 6]

A company that owns many restaurants wants to determine if there are differences in the quality of the food cooked for three different meals: breakfast, lunch and dinner.

Their quality assurance team randomly selects 500 items of food to inspect. The quality of this food is classified as perfect, satisfactory, or poor. The data is summarized in the following table.

		Quality			
		Perfect	**Satisfactory**	**Poor**	**Total**
Meal	**Breakfast**	101	124	7	**232**
	Lunch	68	81	5	**154**
	Dinner	35	69	10	**114**
	Total	**204**	**274**	**22**	**500**

An item of food is chosen at random from these 500.

(a) Find the probability that its quality is not perfect, given that it is from breakfast. [2]

A χ^2 test at the 5% significance level is carried out to determine if there is significant evidence of a difference in the quality of the food cooked for the three meals.

The critical value for this test is 9.488.

The hypotheses for this test are:

H_0: The quality of the food and the type of meal are independent.
H_1: The quality of the food and the type of meal are not independent.

(b) Find the χ^2 statistic. [2]

(c) State, with justification, the conclusion for this test. [2]

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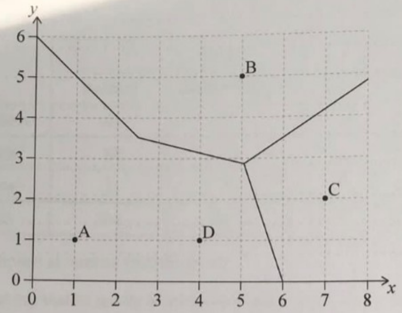
7. [Maximum mark: 6]

Ani owns four cafes represented by points A, B, C and D. Ani wants to divide the area into delivery regions. This process has been started in the following incomplete Voronoi diagram, where 1 unit represents 1 kilometre.



The midpoint of CD is $(5.5, 1.5)$.

(a) Show that the equation of the perpendicular bisector of [CD] is $y = -3x + 18$. [3]

(b) Complete the Voronoi diagram shown above. [1]

Ani opens an office equidistant from three of the cafes, B, C and D. The equation of the perpendicular bisector of [BC] is $3y = 2x - 1.5$.

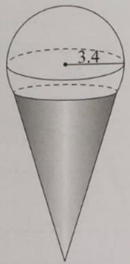
(c) Find the coordinates of the office. [2]

(This question continues on the following page)

8. [Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\,\text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\,\text{cm}$ and a radius of $3\,\text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

9. [Maximum mark: 6]

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of $4\,\text{cm}$ and a standard deviation of $0.25\,\text{cm}$.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than $3.7\,\text{cm}$. [2]

It is known that 30% of the seeds have a length greater than $k\,\text{cm}$.

(b) Find the value of k. [2]

For a seed of length $d\,\text{cm}$, chosen at random, $\text{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

10. [Maximum mark: 8]

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5\,,\ t \geq 0\,,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

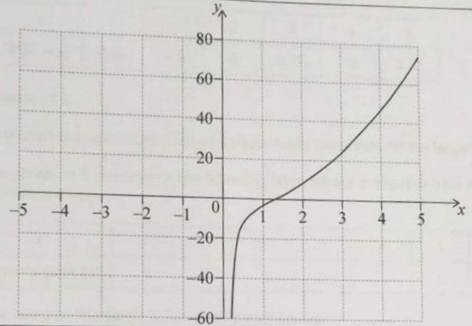
Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

(d) Write down two limitations of using $h(t)$ to model the height of the basketball. [2]

11. [Maximum mark: 7]

Consider $f(x) = 3x^2 - \frac{5}{x}$, $x \neq 0$. The graph of f for $0 < x \leq 5$ is shown on the following axes.



(a) (i) Sketch the graph of f, for $-5 \leq x < 0$, on the same axes.

(ii) Write down the x-coordinate of the local minimum point. [4]

(b) Use your graphic display calculator to find the solutions to the equation $f(x) = 20$. [2]

(c) Write down the equation of the vertical asymptote for the graph of f. [1]

12. [Maximum mark: 5]

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathrm{P}(X=x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

The player has a chance to win money based on how many times they hit the target.

The gain for the player, in \$, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain (\$)	–4	–3	–1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]